

Majorana Bound States in Proximity Junctions of Superconducting Nanowires with Dresselhaus Spin-orbit Coupling

Satoshi Ikegaya¹, Yasuhiro Asano^{1,2,4}, and Yukio Tanaka^{3,4}

¹*Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan*

²*Center of Topological Science and Technology, Hokkaido University, Sapporo 060-8628, Japan*

³*Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan*

⁴*Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia*

(Dated: October 15, 2014)

We theoretically study transport properties of nanowires with the Dresselhaus [110] spin-orbit coupling under the in-plane Zeeman potential and the proximity-induced s -wave pair potential. In the topologically nontrivial phase, the nanowire hosts the Majorana fermions at its edges and the number of the Majorana bound states is equal to the number propagating channels (N_c). When we attach a normal metal to the superconductor, such Majorana bound states penetrate into the dirty normal segment and form the N_c resonant transmission channels there. We show that chiral symmetry of the electronic states protects the Majorana bound states at the zero energy even in the presence of impurities. As a result, we find that the zero-bias conductance of normal-nanowire / superconducting-nanowire junctions is quantized at $2e^2N_c/h$ independent of the random potentials.

PACS numbers: Valid PACS appear here

I. INTRODUCTION

Majorana Fermion, particle which is own antiparticle, was originally predicted by Ettore Majorana in high energy physics¹. Recently, however, physics of Majorana fermion has been a hot issue in condensed matter physics since the emergence of Majorana fermion was pointed out at surfaces of topologically nontrivial superconductors². Detecting a Majorana Fermion and controlling of Majorana bound states (MBSs) have been a desired subject to realize the fault-tolerant topological quantum computation^{3,4}. There are several suggested systems hosting MBSs such as p wave superconductors^{5,6}, topological insulator/superconductor heterostructures⁷, semiconductor/superconductor junctions with strong spin-orbit interaction^{8–13}, helical superconductors¹⁴, and superconducting topological insulators¹⁵. The most practical system among them is a semiconductor nanowire fabricated on top of a superconductor because of its controllability for the emergence of MBS by changing the chemical potential in the nanowire and by applying the Zeeman field onto it^{11–13}. The coexistence of the Rashba spin-orbit coupling and the Zeeman potential enables a topologically nontrivial superconducting state in the nanowire in the presence of proximity induced pair potential there. Even so, it is still very difficult to demonstrate convincing evidences of Majorana fermions in experiments^{16–18} because we need to tune the number of propagating channels in nanowires N_c . The N_c should be unity^{11,12} when the Zeeman field is parallel to the nanowire. Alternatively, N_c should be odd integer numbers when the Zeeman field is applied perpendicular direction to the nanowire¹⁶. In the latter case, the Zeeman field may destroy the pair potential.

In such situation, we seek an alternative way of realizing Majorana fermion by tuning the spin-orbit interactions. The Dresselhaus spin-orbit interactions are caused

by breaking the lattice inversion symmetry¹⁹. In InSb or GaAs, for example, the Dresselhaus [110] spin-orbit interactions can be large on their film growing along the [110] direction. A theoretical study has shown that such artificial superconductor hosts the dispersionless surface Andreev bound states which is nothing other than the MBSs^{10,20}. To have topologically nontrivial superconducting state, the Zeeman field should be applied in plane, which is an advantage of this method. The [110] Dresselhaus nanowire superconductor is unitary equivalent to the two-dimensional 'polar state' in ³He²¹. It has been well known that the polar state has surface Andreev bound states^{22,23} as a result of the sign change of the pair potential on the Fermi surface^{24,25}. Today such surface states are recognized as the topologically protected edge states reflecting the topologically nontrivial character of the superconducting phase²⁶. Although two of authors have reported the anomalous proximity effect of superconductors in the polar state^{27–31}, such superconducting state has never been experimentally confirmed in any compounds. This paper suggests a way of artificially realizing the 2D polar superconductor by combining existing materials. The anomalous proximity effect in the Dresselhaus nanowire superconducting junctions is strongly related to the physics of the odd-frequency Cooper pairs³².

In this paper, we theoretically study the transport properties of nanowires with strong Dresselhaus [110] spin-orbit interaction by using the lattice Green function method on the two-dimensional tight-binding lattice. We first calculate the local density of states (LDOS) at the edge of the semi-infinite nanowire. The Dresselhaus [110] nanowire with in-plane magnetic field shows the large zero-energy peak independent of N_c . The zero-bias differential conductance in normal-metal/superconductor (NS) junctions on the nanowire shows the quantization at $2e^2N_c/h$ irrespective of the degree of disorder in the

normal segment. We also show the fractional current-phase ($J - \varphi$) relationship in superconductor/normal-metal/superconductor (SNS) junctions on the nanowire. The resonant transmission through the MBS in the normal segment is responsible for such unusual low energy transport in nanowires^{27–30,32,33}. In addition to numerical simulation, we solve the Bogoliubov-de-Gennes (BdG) equation analytically and discuss the stability of MBSs in the Dresselhaus[110] nanowire with in-plane magnetic fields. We find that chiral symmetry of the BdG Hamiltonian protects the MBSs at the zero-energy²⁶. Our results indicate a way of detecting the MBSs in experiments.

This paper is organized as follows. In Sec. II, we compare the local density of states at the edge of the Dresselhaus nanowire superconductors with those of the Rashba nanowire superconductors. The numerical result for the transport properties are also presented. In Sec. III, we discuss the stability of the MBSs based on the analytical solution of the BdG equation. In Sec. IV, effects of disorder on the MBS in the normal metal are discussed. The conclusion is given in Sec. V.

II. NUMERICAL RESULTS

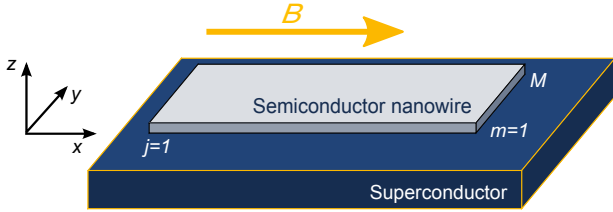


FIG. 1. (Color online) Schematic picture of a semiconductor nanowire proximity coupled to a s -wave superconductor.

A. Local Density of States

Let us consider a nanowire with the strong spin-orbit coupling fabricated on a metallic superconductor as shown in Fig. 1. The nanowire is in the superconducting state due to the proximity-induced s -wave pair potential. The thickness of nanowire is sufficiently small so that only the lowest subband in the z direction for each spin degree of freedom is occupied. We describe the present nanowire by using the tight-binding model in two-dimension. A lattice site is pointed by a vector $\mathbf{r} = j\mathbf{x} + m\mathbf{y}$, where \mathbf{x} and \mathbf{y} are the unit vectors in the x and the y directions, respectively. We consider the nanowire as the semi-infinite system in the x direction (i. e., $1 \leq j \leq \infty$). In the y direction, the number of the lattice site is M and the hard-wall boundary condition is

applied. The nanowire is described by the Bogoliubov-de-Gennes(BdG) Hamiltonian,

$$H_{\text{BdG}} = H_{\text{kin}} + H_Z + H_D^{110} + H_{\Delta}, \quad (1)$$

$$\begin{aligned} \hat{H}_{\text{kin}} = & -t \sum_{\mathbf{r}, \sigma} \sum_{\mathbf{R}=\mathbf{x}, \mathbf{y}} \left(c_{\mathbf{r}+\mathbf{R}, \sigma}^{\dagger} c_{\mathbf{r}, \sigma} + c_{\mathbf{r}, \sigma}^{\dagger} c_{\mathbf{r}+\mathbf{R}, \sigma} \right) \\ & + \sum_{\mathbf{r}, \sigma} (4t - \mu) c_{\mathbf{r}, \sigma}^{\dagger} c_{\mathbf{r}, \sigma}, \end{aligned} \quad (2)$$

$$H_Z = - \sum_{\mathbf{r}, \alpha, \beta} V_{ex} (\sigma_1)_{\alpha, \beta} c_{\mathbf{r}, \alpha}^{\dagger} c_{\mathbf{r}, \beta}, \quad (3)$$

$$H_D^{110} = -i \frac{\lambda_D}{2} \sum_{\mathbf{r}, \alpha, \beta} (\sigma_3)_{\alpha, \beta} \left(c_{\mathbf{r}+\mathbf{x}, \alpha}^{\dagger} c_{\mathbf{r}, \beta} - c_{\mathbf{r}, \alpha}^{\dagger} c_{\mathbf{r}+\mathbf{x}, \beta} \right), \quad (4)$$

$$H_{\Delta} = \sum_{\mathbf{r}} \Delta_0 \left(c_{\mathbf{r}, \uparrow}^{\dagger} c_{\mathbf{r}, \downarrow}^{\dagger} + H.c. \right), \quad (5)$$

where $c_{\mathbf{r}, \sigma}^{\dagger} (c_{\mathbf{r}, \sigma})$ is the creation(annihilation) operator of an electron at the site \mathbf{r} with spin $\sigma = (\uparrow \text{ or } \downarrow)$, t denotes the hopping integral, μ is the chemical potential, λ_D represents the strength of the Dresselhaus [110] spin-orbit interaction, Δ_0 is the proximity-induced s -wave pair potential at the zero temperature. The Pauli's matrices in spin space are represented by $\hat{\sigma}_j$ for $j = 1 - 3$ and the unit matrix in spin space is $\hat{\sigma}_0$. By tuning the magnetic field B in the x direction as shown in Fig. 1, it is possible to introduce the external Zeeman potential V_{ex} . We measure the energy and the length in the units of t and lattice constant, respectively. Throughout this paper, we fix several parameters as $\mu = 1.0t$, $\lambda_D = 0.2t$ and $\Delta_0 = 0.1t$.

At first, we focus on the local density of states (LDOS) at the edge of the nanowire. The LDOS averaged over M lattice sites in the y direction is defined by

$$\rho(j, E) = -\frac{1}{\pi M} \sum_m \text{Im} \left[\text{Tr} \{ \hat{G}(\mathbf{r}, \mathbf{r}, E + i\delta) \} \right], \quad (6)$$

where $\hat{G}(\mathbf{r}, \mathbf{r}, E)$ is the normal Green's function at the site \mathbf{r} with the energy E measured from the Fermi energy, and Tr represents the trace in spin space. To calculate the LDOS, we add the small imaginary part $i\delta$ to the energy in the Green's function. We calculate the Green's function by using the lattice Green's function method^{34,35}. In Fig. 2(a) and (b), we plot the LDOS at $j = 1$ as a function of the energy. The width of system M is chosen as 10. The results are normalized by the density of states at the Fermi energy in the clean normal nanowire N_0 . The Amplitude of Zeeman potential V_{ex} is $1.2t$ and $1.5t$ in Fig. 2(a) and (b), respectively. As a result, the number of propagating channels N_c , is 5 and 6 in Fig. 2(a) and (b), respectively. The LDOS at the edge of the Dresselhaus nanowires shows the single zero-energy peak irrespective of N_c . This is a robust feature appearing as far as the Zeeman potential V_{ex} is larger than a critical value $V_c = 0.92t$. The critical value of the Zeeman potential is discussed in Sec.III C.

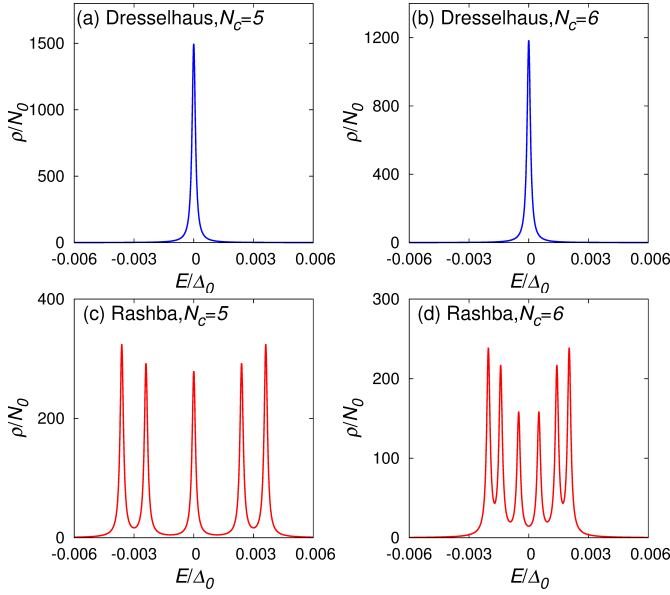


FIG. 2. (Color online) The LDOS at the surface ($j = 1$) of superconductors is plotted as a function of the energy. (a) Dresselhaus [110] nanowire for $V_{ex} = 1.2t$ ($N_c = 5$). (b) Dresselhaus [110] nanowire for $V_{ex} = 1.5t$ ($N_c = 6$). (c) Rashba nanowire for $V_{ex} = 1.2t$ ($N_c = 5$). (d) Rashba nanowire for $V_{ex} = 1.5t$ ($N_c = 6$). The small imaginary part of energy in the Green's function $i\delta$ is chosen as $i10^{-4}\Delta_0$.

For comparison, we also plot the results for the nanowire with Rashba spin-orbit coupling in Fig. 2(c) and (d). To describe the Rashba nanowires, we replace the H_D^{110} by

$$H_R = -i\frac{\lambda_R}{2} \sum_{\alpha,\beta} \sum_{\mathbf{r}} \left[(\sigma_2)_{\alpha,\beta} \left(c_{\mathbf{r}+\mathbf{x},\alpha}^\dagger c_{\mathbf{r},\beta} - c_{\mathbf{r},\alpha}^\dagger c_{\mathbf{r}+\mathbf{x},\beta} \right) - (\sigma_1)_{\alpha,\beta} \left(c_{\mathbf{r}+\mathbf{y},\alpha}^\dagger c_{\mathbf{r},\beta} - c_{\mathbf{r},\alpha}^\dagger c_{\mathbf{r}+\mathbf{y},\beta} \right) \right], \quad (7)$$

and replace H_Z by

$$H'_Z = - \sum_{\mathbf{r},\alpha,\beta} V_{ex} (\sigma_3)_{\alpha,\beta} c_{\mathbf{r},\alpha}^\dagger c_{\mathbf{r},\beta}. \quad (8)$$

representing the magnetic field in the z direction. We chose V_{ex} as $1.2t$ resulting $N_c = 5$ in Fig. 2(c) and $V_{ex} = 1.5t$ resulting $N_c = 6$ in (d), where $\lambda_R = 0.2t$ and $M = 10$. As already discussed in Ref. 16, the LDOS of the Rashba nanowire shows the zero-energy peak only when N_c is odd integer numbers. In addition, the number of peaks in the subgap energy window is equal to N_c . Therefore, in the Rashba nanowires, we need the delicate tuning of the wire width and the Zeeman field to have the MBS indicated by the zero-energy peak.

In Fig. 3(a), we plot the LDOS with $E = 0$ at the edge of superconductors as a function of the wire width M for $V_{ex} = 1.2t$. The LDOS for the Dresselhaus nanowire is almost constant independent of M . Namely, the number of the zero-energy states at the edge increases proportionally to M . In Fig. 3 (b), we also plot the results for the

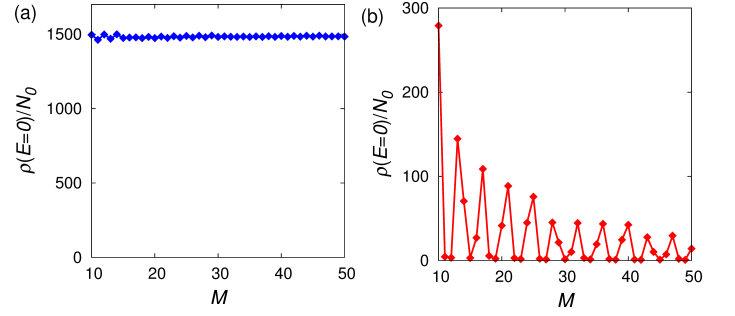


FIG. 3. (Color online) The LDOS at the zero-energy is plotted as a function of the wire width M . The Zeeman potential V_{ex} is chosen as $1.2t$. The results for the Dresselhaus nanowire and for the Rashba nanowire are plotted in (a) and (b), respectively. The small imaginary part for the Green's function $i\delta$ is chosen as $i10^{-4}\Delta_0$.

Rashba nanowire. The LDOS for the Rashba nanowire takes the zero and the nonzero values alternatively as a function of M . The envelop function of its amplitude gradually decreases with increasing M , which reflects a fact that the number of the zero-energy states is at most unity in the Rashba case.

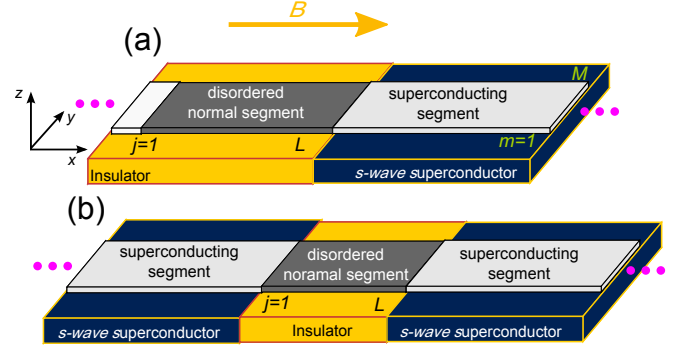


FIG. 4. (Color online) Schematic pictures of (a) NS and (b) SNS junctions.

B. Conductance

Secondly we study the conductance in the NS junctions of nanowire superconductors as shown in Fig. 4(a). A nanowire is fabricated on an insulator/metallic superconductor junction. The segment on the insulator and that on the superconductor are in the normal and the superconducting state, respectively. The present junction consists three segments: an ideal lead wire ($-\infty \leq j \leq 0$), a normal disordered segment ($1 \leq j \leq L$) and a superconducting segment ($L+1 \leq j \leq \infty$). The superconducting segment is described by the BdG Hamiltonian in Eq. (1). The normal-metal segment ($-\infty \leq j \leq L$) is described by Eq. (1) by setting the pair potential Δ_0 to zero. In addition, we introduce the potential disorder in $1 \leq j \leq L$

by

$$H_{\text{imp}} = \sum_{\sigma} \sum_{1 \leq j \leq L, m} V_0(\mathbf{r}) c_{\mathbf{r}, \sigma}^{\dagger} c_{\mathbf{r}, \sigma}. \quad (9)$$

The amplitude of impurity potentials is given randomly in the range of $-W/2 \leq V_0(\mathbf{r}) \leq W/2$. We calculate the differential conductance G_{NS} of the NS junctions based on the Blonder-Tinkham-Klapwijk formula³⁶

$$G_{\text{NS}}(eV) = \frac{e^2}{h} \sum_{\zeta, \eta} \left[\delta_{\zeta, \eta} - |r_{\zeta, \eta}^{ee}|^2 + |r_{\zeta, \eta}^{he}|^2 \right]_{eV=E}, \quad (10)$$

where $r_{\zeta, \eta}^{ee}$ and $r_{\zeta, \eta}^{he}$ denote the normal and Andreev reflection coefficients at the energy E , respectively. The index ζ and η label the outgoing channel and the incoming one, respectively. These reflection coefficients are calculated by using the lattice Green's function method^{34,35}. In Fig. 5(a), we present the differential conductance of the Dresselhaus nanowires as a function of the bias voltage for several choices of the length of the disordered segments L , where we choose the parameters as $M = 10$, $W = 2.0t$, and $V_{ex} = 1.2t$. In the present parameter choice, N_c becomes 5. The results are the normalized to $G_Q = 2e^2/h$. The differential conductance decreases with increasing L for the finite bias voltage. However, the zero-bias conductance is quantized at $G_Q N_c$ irrespective of L . The results suggest that the perfect transmission channels exist in the disordered segment²⁸ and their number is equal to N_c .

For comparison, we also plot the results for the Rashba nanowire in Fig. 5(b). The results show that the zero-bias conductance is not quantized and decrease with increasing L . Even if a MBS appears at the zero-energy, its contribution to the zero-bias conductance is relatively small. Therefore it is difficult to demonstrate the presence of MF by the conductance measurement in experiments.

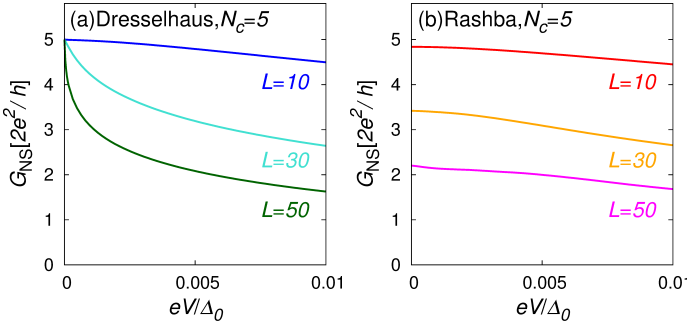


FIG. 5. (Color online) The differential conductance is plotted as a function of the bias voltage for several choices of the length of disordered segment L , where $W = 2.0t$, $V_{ex} = 1.2t$, and $M = 10$. The results of the Dresselhaus nanowire and those of the Rashba nanowire are plotted in (a) and (b), respectively. The number of propagating channels N_c is 5 in both (a) and (b). The number of samples used for the random ensemble average is 10^3 .

In Fig. 6, we plot the zero-bias conductance as a function of the width of nanowire M for $V_{ex} = 1.2t$. The

length of disordered segment L is chosen as 30 and 50. In the Dresselhaus nanowires, the perfect quantization of the zero-bias conductance can be seen irrespective of the width of the nanowire M . This result reflects the presence of a MBS for each propagating channel. The presence of MBSs can be checked by the quantized value of the zero-bias conductance. In the case of the Rashba nanowire, on the other hand, the zero-bias conductance slowly decreases with increasing M as shown in Fig. 6. The results are away from the $2e^2 N_c/h$ for all M .

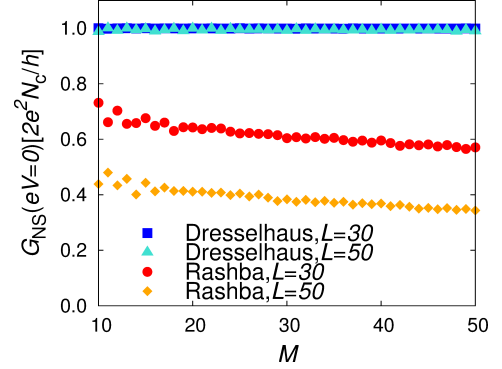


FIG. 6. (Color online) The zero-bias conductance is plotted as a function of the wire width M for $L = 30$ and 50. The results are normalized to $G_Q N_c$. The number of samples used for the random ensemble average is 10^3 .

C. Josephson Current

Finally, we study the Josephson effect in the junction shown in Fig. 4(b). The junction consists three segment: a disordered normal segment ($1 \leq j \leq L$) and two Dresselhaus superconducting nanowires ($\infty \leq j \leq 0$ and $L+1 \leq j \leq \infty$). The pair potential for the left superconductor is described by

$$H_{\Delta}^L = \sum_{j \leq 0, m} \left(\Delta_0 e^{i\varphi} c_{\mathbf{r}, \uparrow}^{\dagger} c_{\mathbf{r}, \downarrow}^{\dagger} + H.c. \right), \quad (11)$$

where φ corresponds to the phase difference of the pair potential between two superconductors. The Josephson current is calculated by using the Green's function method³⁷. In Fig. 7, we plot the Josephson current at $T = 0.001T_c$ as a function of the phase difference for several choices of W such as $1.0t$, $2.0t$ and $3.0t$, where $M = 10$, $L = 30$, and $V_{ex} = 1.5t$. For comparison, we also plot the result for the conventional s -wave superconductor junction (i.e., $V_{ex} = 0$ and $\lambda_D = 0$) with a dashed line. The current-phase relationship for the conventional junction slightly deviate the sinusoidal function. The results suggest the small contribution of the higher harmonics such as $\sin(2\varphi)$ and $\sin(3\varphi)$ to the Josephson current. This is the well know behavior of the Josephson current in diffusive SNS junction of the metallic superconductor³⁸.

On the other hand, the Josephson current in the Dresselhaus nanowires indicates the large contribution of the higher harmonics to the Josephson current at a low temperature. As a consequence, the results are close to the fractional current-phase relationship of $J \propto \sin(\varphi/2)$ irrespective of W . Such fractional relationship also indicates the perfect transmission through the disordered normal segment^{29,32}.

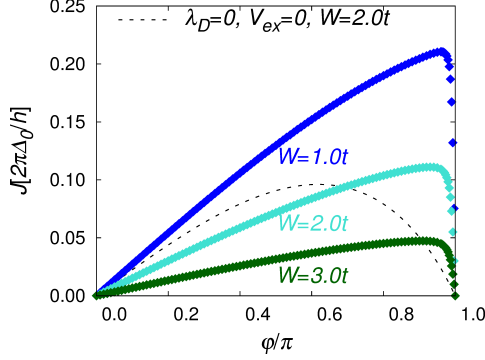


FIG. 7. (Color online) The Josephson current at $T = 0.001T_c$ is plotted as a function of the phase difference, where $L = 30$, $M = 10$, and $V_{ex} = 1.5t$. We plot the result for several choices of W such as $1.0t$, $2.0t$ and $3.0t$. We also plot the results for the conventional s -wave superconductor (i.e., $V_{ex} = 0$ and $\lambda_D = 0$) at $W = 2.0t$ with a dashed line. The number of samples used for the random ensemble average is 500 for all the plots.

III. SURFACE MAJORANA BOUND STATES

In this section, we analyze the properties of the Majorana bound states appearing at the edge of the Dresselhaus nanowire as shown in Fig. 1.

A. Wave function of zero-energy edge states

Here we consider the nanowire in the continuous space for simplicity. The BdG Hamiltonian of the Dresselhaus nanowire is represented by

$$\tilde{H}_0 = \begin{bmatrix} \hat{h} & i\Delta_0\hat{\sigma}_2 \\ -i\Delta_0\hat{\sigma}_2 & -\hat{h}^* \end{bmatrix}, \quad (12)$$

$$\hat{h} = \xi\hat{\sigma}_0 - V_{ex}\hat{\sigma}_1 + i\lambda_D\partial_x\hat{\sigma}_3, \quad \xi = \frac{\hbar^2}{2m}\nabla^2 - \mu, \quad (13)$$

where m denotes the effective mass of an electron. In what follows, we assume large enough Zeeman potential so that $\lambda_D k_F \ll V_{ex}$ is satisfied with $k_F = \sqrt{2m\mu}/\hbar$. By applying the unitary transformations in Appendix B, we

obtain the deformed BdG Hamiltonian

$$\tilde{H}_{\text{eff}} = \tilde{H}_{p_x} + \tilde{V}_\Delta + O\left[\left(\frac{\lambda_D k_F}{V_{ex}}\right)^2\right], \quad (14)$$

$$\tilde{H}_{p_x} = \begin{bmatrix} \hat{h}_{p_x,\uparrow} & 0 \\ 0 & \hat{h}_{p_x,\downarrow} \end{bmatrix}, \quad (15)$$

$$\hat{h}_{p_x,\sigma} = \begin{bmatrix} \xi + s_s V_{ex} & -s_s \frac{\lambda_D \Delta_0}{V_{ex}} \partial_x \\ s_s \frac{\lambda_D \Delta_0}{V_{ex}} \partial_x & -\xi - s_s V_{ex} \end{bmatrix}, \quad (16)$$

$$\tilde{V}_\Delta = \begin{bmatrix} 0 & i\Delta_0\hat{\sigma}_2 \\ -i\Delta_0\hat{\sigma}_2 & 0 \end{bmatrix}, \quad (17)$$

$$s_s = \begin{cases} 1 & \text{for } \sigma = \uparrow \\ -1 & \text{for } \sigma = \downarrow \end{cases}. \quad (18)$$

Since $\lambda_D k_F \ll V_{ex}$, we ignore the higher order terms indicated by $O[(\lambda_D k_F/V_{ex})^2]$ in the argument below. The diagonal components $\hat{h}_{p_x,\sigma}$ are equivalent to the Hamiltonian of the spin-triplet p_x -wave superconductor. Thus the BdG Hamiltonian represents the spin-full p_x -wave superconductor \tilde{H}_{p_x} with the spin-mixing term \tilde{V}_Δ .

We first solve the BdG equation for the zero-energy edge states by neglecting the spin-mixing term \tilde{V}_Δ . We will discuss the effects of \tilde{V}_Δ later on. The BdG equation at the zero-energy reads

$$\tilde{H}_{p_x}\varphi_{\nu_0}(\mathbf{r}) = 0. \quad (19)$$

where

$$\varphi_{\nu_0}(\mathbf{r}) \equiv [u_{\nu_0,\uparrow}(\mathbf{r}), v_{\nu_0,\uparrow}(\mathbf{r}), u_{\nu_0,\downarrow}(\mathbf{r}), v_{\nu_0,\downarrow}(\mathbf{r})]^T$$

is the eigen wave function of the zero-energy states labeled by ν_0 . Under the hard-wall boundary condition in the y direction, the wave function is represented as

$$\varphi_{\nu_0}(\mathbf{r}) = \sqrt{\frac{2}{M}} \sum_n \varphi_n(x) \sin\left(\frac{n\pi}{M}y\right), \quad (20)$$

where n indicates the transmission channel in the nanowires and $\varphi_n(x)$ is a vector with the four components. In the x direction, we assume that the length of the nanowire is $2L$ (i.e., $-L \leq x \leq L$) and we apply the hard-wall boundary conditions at the edge of the nanowire,

$$\varphi_n(-L) = \varphi_n(L) = 0. \quad (21)$$

We show how to solve the BdG equation in Appendix B. Here we summarize the results and discuss important properties of the solution. The BdG equation can be solved for each transmission channel indicated by n in which

$$\mu_n = \mu - \frac{\hbar^2}{2m} \left(\frac{n\pi}{M}\right)^2 \quad (22)$$

represents the effective chemical potential. When $\mu_n < -V_{ex}$, there is no solution of Eq. (19) with Eq. (21). For

$\mu_n > V_{ex}$, we can find the two solutions for each transmission channel at each edge: one is in the spin-up sector and the other is in the spin-down one in Eq. (15). At the edge around $x = L$, for example, two zero-energy states in the two spin sectors are degenerate. However such doubly-degenerate zero-energy states are unstable in the presence of the \tilde{V}_Δ because the spin-mixing terms hybridize the two zero-energy states and lift the degeneracy. Finally, for $-V_{ex} < \mu_n < V_{ex}$, we obtain the only one zero-energy edge state for each transmission channel at each edge. Namely the zero-energy state in spin-up sector disappears and only the zero-energy state in the spin-down sector remains at each edge. Since the up-spin state is absent, \tilde{V}_Δ does not affect such zero-energy edge states. The wave function at the left edge $\varphi_n^L(x)$ and that at the right edge $\varphi_n^R(x)$ can be represented as

$$\varphi_n^L(x) = C_n^L \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \sin \left[\sqrt{k_{n,\downarrow}^2 - \xi_D^{-2}}(x + L) \right] e^{-x/\xi_D}, \quad (23)$$

$$\varphi_n^R(x) = C_n^R \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \sin \left[\sqrt{k_{n,\downarrow}^2 - \xi_D^{-2}}(x - L) \right] e^{x/\xi_D}, \quad (24)$$

where

$$\xi_D = \frac{\hbar^2 V_{ex}}{m \lambda_D \Delta_0}, \quad k_{n,\sigma} = \frac{\sqrt{\mu_n - s_s V_{ex}}}{\hbar}, \quad (25)$$

with C_n^L and C_n^R being the normalization coefficients. When $L/\xi_D \gg 1$, the two zero-energy states localizing at $x = \pm L$ are decoupled from each other. The number of the zero-energy states at each edge is equal to the number of channels which satisfies $-V_{ex} < \mu_n < V_{ex}$.

B. Stability of zero-energy states

The BdG Hamiltonian in Eq. (14) preserves chiral symmetry,

$$\Gamma \tilde{H}_{\text{eff}} \Gamma^{-1} = -\tilde{H}_{\text{eff}}, \quad \Gamma = \begin{bmatrix} \hat{\sigma}_1 & 0 \\ 0 & \hat{\sigma}_1 \end{bmatrix}. \quad (26)$$

In the presence of chiral symmetry, it is possible to introduce the eigen states of \tilde{H}_{eff}^2

$$\tilde{H}_{\text{eff}}^2 \chi_\lambda(\mathbf{r}) = E^2 \chi_\lambda(\mathbf{r}), \quad (27)$$

where $\chi_\lambda(\mathbf{r})$ is also the eigen states for Γ

$$\Gamma \chi_\lambda(\mathbf{r}) = \lambda \chi_\lambda(\mathbf{r}). \quad (28)$$

The eigenvalue λ is either +1 or -1. See also Appendix A for details. By using these eigen states $\chi_\lambda(\mathbf{r})$, the states

belonging to the zero-energy $\varphi_{\nu_0}(\mathbf{r})$ can be represented by²⁶

$$\varphi_{\nu_0 \lambda}(\mathbf{r}) = \chi_{\nu_0 \lambda}(\mathbf{r}), \quad (29)$$

where $\chi_{\nu_0 \lambda}(\mathbf{r})$ satisfies

$$\tilde{H}_{\text{eff}}^2 \chi_{\nu_0 \lambda}(\mathbf{r}) = 0. \quad (30)$$

The index ν_0 labels the eigen states belonging to the zero energy. From the results above, we conclude that $\varphi_{\nu_0 \lambda}(\mathbf{r})$ is the eigen state of Γ belonging to λ , which is an important fact leading to the stability of the zero-energy states.

The situation in the nonzero-energy states is different that in the zero-energy states. As shown in Appendix A, the nonzero-energy states are always described by the linear combination of two states: one belongs to $\lambda = 1$ (i.e., $\chi_+(\mathbf{r})$) and the other belongs to $\lambda = -1$ (i.e., $\chi_-(\mathbf{r})$). Generally speaking, perturbations may lift zero-energy states to nonzero-energy ones. Such modification happens only when the perturbations couple the two zero-energy states belonging to opposite λ as schematically illustrated in Fig. 8(a) and (b). This argument is valid as far as the perturbations preserve chiral symmetry. Therefore the zero-energy states belong to the same eigenvalue of λ are stable and remain at the zero-energy when perturbations preserve chiral symmetry in Eq. (26). In the Dresselhaus nanowires for $-V_{ex} < \mu_n < V_{ex}$, it is easy to confirm that φ_n^L in Eq. (23) belongs to $\lambda = -1$, whereas φ_n^R in Eq. (24) belongs to $\lambda = 1$. Since they are spatially separated, the zero-energy states at the two edges are robust under perturbations preserving chiral symmetry as illustrated in Fig. 8(c).

Finally, we note that chiral symmetry in the original basis represented by

$$\tilde{\Gamma}_0 \tilde{H}_0 \tilde{\Gamma}_0^{-1} = -\tilde{H}_0, \quad (31)$$

$$\Gamma_0 = \begin{bmatrix} 0 & -i\hat{\sigma}_1 \\ i\hat{\sigma}_1 & 0 \end{bmatrix}, \quad (32)$$

where \tilde{H}_0 is the original Hamiltonian in Eq. (12). This fact implies that the zero-energy states are robust under perturbations preserving chiral symmetry even if we take the higher order terms of $(\lambda_D k_F / V_{ex})$ into account.

C. Critical value of Zeeman potential

The anomalous properties in the low energy transport in Sec. II appear when the Zeeman field is larger than a critical value (i.e., $V_{ex} > V_c$). Here we discuss why the low energy spectra of the edge states drastically changes at $V_{ex} = V_c$. We consider the periodic boundary condition in the x direction to obtain the momentum representation of \tilde{H}_{eff} in Eq (14), The state vectors can be written as

$$\varphi(\mathbf{r}) = \sqrt{\frac{2}{LM}} \sum_{n,k} \varphi_{k,n} e^{ikx} \sin\left(\frac{n\pi}{M}y\right), \quad (33)$$

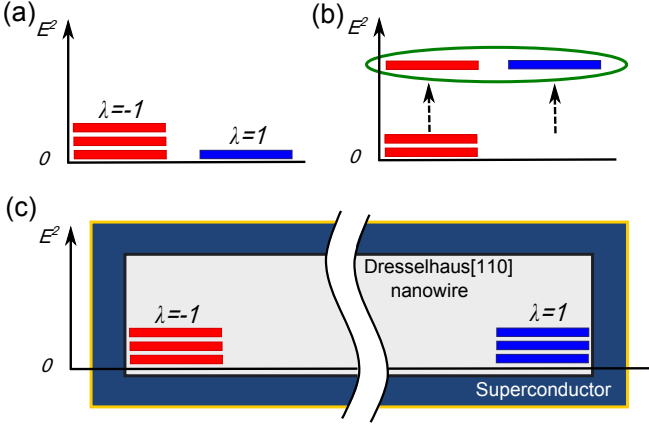


FIG. 8. (Color online) (a) Four zero-energy states are illustrated. The three of them belong to $\lambda = -1$ and the one remaining state belongs to $\lambda = 1$. (b) Two states with opposite λ are coupled with each other and form the non-zero energy states. The remaining two states stay at the zero-energy because they do not have the coupling partners. (c) The schematic picture of the zero-energy edge states of the Dresselhaus nanowire. The number of the zero-energy states at either edge is equal to N_c . All of the edge states at the left (right) belong to $\lambda = -1$ ($\lambda = 1$).

where n labels the channels in the y direction and k denotes the wave number in the x direction. The Hamiltonian in Eq. (16) in the momentum space is represented by

$$\hat{h}_{p_x, \sigma}(k, n) = \begin{bmatrix} \xi_n(k) + s_s V_{ex} & -s_s i \frac{\lambda_D \Delta_0}{V_{ex}} k \\ s_s i \frac{\lambda_D \Delta_0}{V_{ex}} k & -\xi_n(k) - s_s V_{ex} \end{bmatrix}, \quad (34)$$

$$\xi_n(k) = \frac{\hbar^2 k^2}{2m} - \mu_n, \quad (35)$$

with μ_n being represented in Eq. (22). As discussed in Sec. III B, the zero-energy edge states are stable when

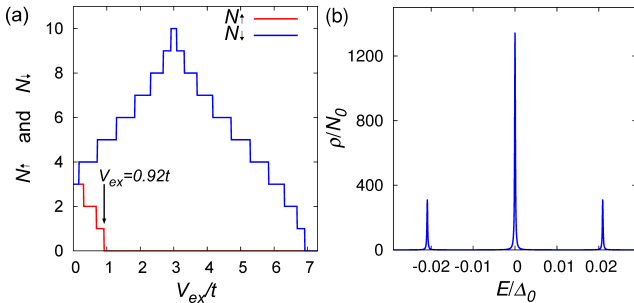


FIG. 9. (Color online) (a) The number of propagating channels N_\uparrow and N_\downarrow are plotted as a function of the Zeeman potential V_{ex} at $M = 10$. N_\uparrow becomes zero at $V_{ex} = 0.92t$. (b) We plot the local density of states at the edge of the Dresselhaus nanowire as a function of energy. The Zeeman potential V_{ex} is chosen as $0.8t$ (i.e., $V_{ex} < 0.92t$). The small imaginary part in the Green's function $i\delta$ is chosen as $i\Delta_0 \times 10^{-4}$.

the condition

$$-V_{ex} < \mu_n < V_{ex} \quad (36)$$

is satisfied. This condition corresponds to the situation in which the dispersion of spin-down sector $\xi_n(k) - V_{ex}$ remains at the Fermi level and the dispersion of spin-up sector leaves away from the Fermi level (i.e., $\xi_n(k) + V_{ex} > 0$). The number of the zero-energy states becomes equal to the number of the propagating channel N_c in the spin-down sector. In Fig. 9(a), we plot the number of the propagating channels as a function of the Zeeman potential, where $N_{\uparrow(\downarrow)}$ represents the number of the propagating channels in spin-up (spin-down) sector. In the tight binding model, the effective chemical potential μ_n should be replaced by

$$\mu_n = \mu - 2t \left[1 - \cos \left(\frac{n\pi}{M+1} \right) \right]. \quad (37)$$

In the spin-up sector, N_\uparrow becomes zero at $V_{ex} = V_c = 0.92t$ at the present parameter choice.

For $V_{ex} < V_c$, the dispersions in both the spin-up and the spin-down sectors remain at the Fermi level. In Fig. 9(b), we show the LDOS at the edge of the Dresselhaus nanowire for $V_{ex} = 0.8t < V_c$. The resulting channel numbers are $N_\uparrow = 1$ and $N_\downarrow = 5$. The edge states also arise from the two spin sectors. But they are interact with each other due to \tilde{V}_Δ . As a result, the energy of such interacting states leave away from the zero energy. The results of LDOS show that two peaks appear at $E = \pm 0.021\Delta_0$ in addition to the large zero-energy peak. Thus the number of the zero-energy edge states is $N_\downarrow - N_\uparrow$ which is less than the number of propagating channels $N_c = N_\downarrow + N_\uparrow$.

For $V_{ex} > V_c$, the Hamiltonian of the Dresselhaus nanowire becomes unitary equivalent to that of the two-dimensional spinless p_x -wave superconductor. It is possible to realize such polar states by tuning the spin-orbit coupling in an alternative way as shown in Appendix C.

D. Majorana Fermions

The field operator of an electron for the BdG Hamiltonian \tilde{H}_{eff} is described as

$$\Psi(\mathbf{r}) = \sum_{\nu} [\varphi_{\nu}(\mathbf{r})\gamma_{\nu} + \Xi\varphi_{\nu}(\mathbf{r})\gamma_{\nu}^{\dagger}], \quad (38)$$

$$\Xi = \Gamma\mathcal{K}, \quad (39)$$

where γ_{ν}^{\dagger} (γ_{ν}) is the creation (annihilation) operator of the Bogoliubov quasiparticle belonging to E_{ν} and Ξ is the charge conjugation operator with \mathcal{K} representing the complex conjugation. Here we focus on the electron operator at the zero-energy states for $V_{ex} > V_c$. As discussed in Sec. III B, the wave function of the zero-energy states are well characterized by the channel index n and

described by

$$\varphi_n^L(\mathbf{r}) = \sqrt{\frac{1}{M}} \varphi_n^L(x) \sin\left(\frac{n\pi}{M}y\right), \quad (40)$$

$$\varphi_n^R(\mathbf{r}) = \sqrt{\frac{1}{M}} \varphi_n^R(x) \sin\left(\frac{n\pi}{M}y\right), \quad (41)$$

where $\varphi_n^L(\mathbf{r})$ ($\varphi_n^R(\mathbf{r})$) correspond to the left(right) edge states. These wave function satisfy

$$\Gamma \varphi_n^L(\mathbf{r}) = -\varphi_n^L(\mathbf{r}), \quad (42)$$

$$\Gamma \varphi_n^R(\mathbf{r}) = \varphi_n^R(\mathbf{r}), \quad (43)$$

as illustrated in Fig. 8(c). Therefore, the electron operator of the zero-energy state is written as

$$\Psi_n(\mathbf{r}) = i\gamma_n^L(\mathbf{r}) + \gamma_n^R(\mathbf{r}), \quad (44)$$

$$\gamma_n^L(\mathbf{r}) = -i \left[\varphi_n^L(\mathbf{r}) \gamma_{n-} - (\varphi_n^L(\mathbf{r}))^* \gamma_{n-}^\dagger \right], \quad (45)$$

$$\gamma_n^R(\mathbf{r}) = \left[\varphi_n^R(\mathbf{r}) \gamma_{n+} + (\varphi_n^R(\mathbf{r}))^* \gamma_{n+}^\dagger \right], \quad (46)$$

where the field operator $\gamma_n^L(\mathbf{r})$ and $\gamma_n^R(\mathbf{r})$ correspond to the edge state on the left hand side and that on the right hand side, respectively. The operator $\gamma_n^L(\mathbf{r})$ is pure imaginary while $\gamma_n^R(\mathbf{r})$ is real in the present gauge choice. It is easy to show that they satisfy the Majorana relation

$$(\gamma_n^L(\mathbf{r}))^\dagger = \gamma_n^L(\mathbf{r}), \quad (47)$$

$$(\gamma_n^R(\mathbf{r}))^\dagger = \gamma_n^R(\mathbf{r}). \quad (48)$$

We conclude that both $\gamma_n^L(\mathbf{r})$ and $\gamma_n^R(\mathbf{r})$ fields describe the Majorana fermions. The above relations hold for each propagating channel n . Therefore the number of Majorana fermions at each edge is N_\downarrow which is equal to N_c for $V_{ex} > V_c$.

IV. MAJORANA BOUND STATES IN NORMAL METALS

The numerical results in Sec. II show that the Majorana bound states penetrate into the diffusive normal segment and form the resonant transmission channels there. The perfect transmission through such Majorana bound states at the zero energy are responsible for the anomalous transport properties. Here we discuss why the Majorana bound states remain at the zero energy even in the presence of impurity potentials in the normal segment.

A. Wave function in normal segment

We first analyze the wave functions in the normal segment using in Eq. (34). In the absence of impurity potential, the wave function in the normal segment at $E = 0$

is described by

$$\phi_n(x) = \begin{bmatrix} 1 + r_{\uparrow,\uparrow}^{ee} \\ r_{\uparrow,\uparrow}^{he} \\ 0 \\ 0 \end{bmatrix} e^{ik_{n,\uparrow}x} + \begin{bmatrix} 0 \\ 0 \\ 1 + r_{\downarrow,\downarrow}^{ee} \\ r_{\downarrow,\downarrow}^{he} \end{bmatrix} e^{ik_{n,\downarrow}x}, \quad (49)$$

$$k_{n,\sigma} = \sqrt{2m(\mu_n - s_\sigma V_{ex})}/\hbar \quad (50)$$

where $r_{\sigma,\sigma}^{ee}$ is the normal reflection coefficients and $r_{\sigma,\sigma}^{he}$ is the Andreev reflection coefficients. Here we consider the wave function at the channel n . The p_x superconductors causes the perfect Andreev reflection at $E = 0$, which results in $r_{\sigma,\sigma}^{ee} = 0$. At the same time, we obtain $r_{\uparrow,\uparrow}^{he} = 1$ and $r_{\downarrow,\downarrow}^{he} = -1$. We find that the wave function in each spin sector is the eigen state of Γ in Eq. (26). Therefore all states in spin-up (spin-down) sector belong to $\lambda = 1$ ($\lambda = -1$) in the normal segment. This conclusion is also true even when we introduce impurity potential into the normal segment because the impurity potentials preserve chiral symmetry in Eq. (26).

B. Effects of disorder

To describe the disordered NS junctions in Fig. 4(a), we introduce the random potential $V_{\text{imp}}(\mathbf{r})$ in the normal segment, (i.e., $0 < x < L$). The Hamiltonian \hat{h} in Eq. (12) is replaced by

$$\hat{h}^{\text{imp}} = \hat{h} + V_{\text{imp}}(\mathbf{r})\hat{\sigma}_0. \quad (51)$$

The Hamiltonian of the NS junction reads,

$$\tilde{H}_{\text{NS}} = \begin{bmatrix} \hat{h}^{\text{imp}} & i\Delta_0\hat{\sigma}_2\Theta(x-L) \\ -i\Delta_0\hat{\sigma}_2\Theta(x-L) & -[\hat{h}^{\text{imp}}]^* \end{bmatrix}. \quad (52)$$

It is easy to show that H_{NS} satisfies the relations,

$$\Gamma_0 H_{\text{NS}} \Gamma_0^{-1} = -H_{\text{NS}}, \quad \Gamma_0 = \begin{bmatrix} 0 & -i\hat{\sigma}_1 \\ i\hat{\sigma}_1 & 0 \end{bmatrix}. \quad (53)$$

Therefore, all of the zero-energy states at the NS interface keep staying at the zero-energy even in the presence of disorder because all them belong to $\lambda = -1$ as shown in Fig. 8(c). Although the channel index n is no longer a good quantum number under the potential disorder, the number of the zero-energy states is still equal to the number of the propagating channels in the spin-down sector $N_\downarrow = N_c$. This argument can be applied to the zero-energy states penetrating into the normal segment. The resonant transmission channels at the zero energy in the normal segment is protected in the presence of chiral symmetry. This explains the perfect quantization of the zero-bias conductance at $2e^2N_c/h$.

To confirm the argument above, we calculate LDOS in the disordered normal segment. In Fig. 10(a), we plot the LDOS at the center of the disordered segment (i.e., $j = 25$) as a function of the energy, where $L = 50$,

$M = 10$, $W = 2.0t$. The Zeeman potential V_{ex} is chosen as $0.5t$ and $1.2t$. In the case of $V_{ex} < V_c = 0.92t$, the states with $\lambda = 1$ remain at the Fermi level in the normal metal. The random impurity potentials mix the penetrated Majorana bound states with $\lambda = -1$ and the normal states with $\lambda = 1$. As a result, the LDOS in the disordered normal segment for $V_{ex} = 0.5t < V_c$ is almost flat around the zero-energy. When $V_{ex} > V_c$, all of the normal states with $\lambda = 1$ pinch off from the Fermi level. Therefore, penetrated Majorana bound states stably remain at the Fermi level. As a result, the LDOS in the disordered normal segment for $V_{ex} = 1.2t > V_c$ shows the large zero-energy peak reflecting the existence of the penetrated Majorana bound states. It is possible to demonstrate how chiral symmetry protects the zero-energy states in the normal segment by analyzing the details of LDOS at the zero-energy in Fig. 10(a). To do this, we calculate the Green's function around the zero energy. The Hamiltonian satisfies,

$$\Xi_0 H_{NS} \Xi_0^{-1} = -H_{NS}, \quad \Xi_0 = \begin{bmatrix} 0 & \mathcal{K} \hat{\sigma}_0 \\ \mathcal{K} \hat{\sigma}_0 & 0 \end{bmatrix}, \quad (54)$$

where Ξ_0 represents the charge conjugation. When $\phi_\nu(\mathbf{r}) \equiv [u_{\nu,\uparrow}(\mathbf{r}), u_{\nu,\downarrow}(\mathbf{r}), v_{\nu,\uparrow}(\mathbf{r}), v_{\nu,\downarrow}(\mathbf{r})]^T$ is the wave function belonging to E_ν , $\bar{\phi}_\nu(\mathbf{r}) = \Xi_0 \phi_\nu(\mathbf{r})$ is the wave function belonging to $-E_\nu$. Using these wave functions, the Green's function is represented by

$$\check{G}(\mathbf{r}, \mathbf{r}; E) = \sum_\nu \left[\frac{\phi_\nu(\mathbf{r}) \phi_\nu^\dagger(\mathbf{r}')}{E + i\delta - E_\nu} + \frac{\bar{\phi}_\nu(\mathbf{r}) \bar{\phi}_\nu^\dagger(\mathbf{r}')}{E + i\delta + E_\nu} \right]. \quad (55)$$

When we consider $|E| \ll \Delta_0$, the wave functions at the zero energy mainly contribute to the Green's function,

$$\check{G}(\mathbf{r}, \mathbf{r}; E) \approx \sum_{\nu_0} \frac{[\phi_{\nu_0}(\mathbf{r}) \phi_{\nu_0}^\dagger(\mathbf{r}') + \bar{\phi}_{\nu_0}(\mathbf{r}) \bar{\phi}_{\nu_0}^\dagger(\mathbf{r}')]}{E + i\delta}, \quad (56)$$

where ν_0 indicates the states at the zero energy. We can immediately confirm that

$$[H_{NS}^2, \Gamma_0] = [H_{NS}^2, \Xi_0] = [\Gamma_0, \Xi_0] = 0. \quad (57)$$

Therefore, as shown in Appendix A, we find that $\phi_{\nu_0}(\mathbf{r})$ is the eigen vector of Ξ_0 and Γ_0 at the same time. From this fact, it is possible to represent the wave function by the linear combination of following vectors

$$\Phi_{\lambda, \alpha, \nu_0}(\mathbf{r}) = \frac{1}{2} \begin{bmatrix} a_{\lambda, \alpha, \nu_0}(\mathbf{r}) \\ -\lambda \alpha a_{\lambda, \alpha, \nu_0}^*(\mathbf{r}) \\ -i\alpha a_{\lambda, \alpha, \nu_0}^*(\mathbf{r}) \\ i\lambda a_{\lambda, \alpha, \nu_0}(\mathbf{r}) \end{bmatrix}, \quad (58)$$

where $\Phi_{\lambda, \alpha, \nu_0}$ is the eigen function of Γ_0 belonging to $\lambda = \pm 1$ and is also the eigen function of Ξ_0 belonging to $i\alpha$ with $\alpha = \pm 1$. We assume $\int d\mathbf{r} |a_{\lambda, \alpha, \nu_0}(\mathbf{r})|^2 = 1$. By using these vectors, the normal Green's function at the

electron space becomes

$$\begin{aligned} \hat{G}(\mathbf{r}, \mathbf{r}'; E) &= \hat{G}^+(\mathbf{r}, \mathbf{r}'; E) + \hat{G}^-(\mathbf{r}, \mathbf{r}'; E), \\ \hat{G}^\pm(\mathbf{r}, \mathbf{r}'; E) &= \frac{1}{4(E + i\delta)} \sum_{\alpha, \nu_0} \\ &\times \begin{bmatrix} a_{\pm, \alpha, \nu_0}(\mathbf{r}) a_{\pm, \alpha, \nu_0}^*(\mathbf{r}') & \mp \alpha a_{\pm, \alpha, \nu_0}(\mathbf{r}) a_{\pm, \alpha, \nu_0}(\mathbf{r}') \\ \mp \alpha a_{\pm, \alpha, \nu_0}^*(\mathbf{r}) a_{\pm, \alpha, \nu_0}^*(\mathbf{r}') & a_{\pm, \alpha, \nu_0}^*(\mathbf{r}) a_{\pm, \alpha, \nu_0}(\mathbf{r}') \end{bmatrix}. \end{aligned} \quad (59)$$

The normal Green's function \hat{G}^+ (\hat{G}^-) are the Green function derived from the wave function with $\lambda = 1$ ($\lambda = -1$). In the same way, the anomalous Green function is represented as

$$\begin{aligned} \hat{F}(\mathbf{r}, \mathbf{r}'; E) &= \hat{F}^+(\mathbf{r}, \mathbf{r}'; E) + \hat{F}^-(\mathbf{r}, \mathbf{r}'; E), \\ \hat{F}^\pm(\mathbf{r}, \mathbf{r}'; E) &= \frac{i}{4(E + i\delta)} \sum_{\alpha, \nu_0} \\ &\times \begin{bmatrix} \alpha a_{\pm, \alpha, \nu_0}(\mathbf{r}) a_{\pm, \alpha, \nu_0}(\mathbf{r}') & \mp a_{\pm, \alpha, \nu_0}(\mathbf{r}) a_{\pm, \alpha, \nu_0}^*(\mathbf{r}') \\ \mp a_{\pm, \alpha, \nu_0}^*(\mathbf{r}) a_{\pm, \alpha, \nu_0}(\mathbf{r}') & \alpha a_{\pm, \alpha, \nu_0}^*(\mathbf{r}) a_{\pm, \alpha, \nu_0}^*(\mathbf{r}') \end{bmatrix}. \end{aligned} \quad (60)$$

By using the relations,

$$\hat{F}^\pm(\mathbf{r}, \mathbf{r}; E) \hat{\sigma}_1 = \mp i \hat{G}^\pm(\mathbf{r}, \mathbf{r}; E), \quad (61)$$

\hat{G}^\pm is described by \hat{G} and \hat{F} obtained in the numerical simulation,

$$\hat{G}^\pm(\mathbf{r}, \mathbf{r}; E) = \frac{1}{2} \left[\hat{G}(\mathbf{r}, \mathbf{r}; E) \pm i \hat{F}(\mathbf{r}, \mathbf{r}; E) \hat{\sigma}_1 \right]. \quad (62)$$

We can separate the numerical results of LDOS at the zero-energy into two contributions

$$\rho(j, E) = \rho^+(j, E) + \rho^-(j, E), \quad (63)$$

$$\rho^\pm(j, E) = -\frac{1}{\pi M} \sum_m \text{Im} \left[\text{Tr} \{ \hat{G}^\pm(\mathbf{r}, \mathbf{r}; E) \} \right]. \quad (64)$$

In Fig. 10(b) and (c), we plot the LDOS corresponding to the states with $\lambda = 1$ (i.e., $\rho^+(j, 0)$) and with $\lambda = -1$ (i.e., $\rho^-(j, 0)$) as a function of the Zeeman potential V_{ex} , respectively. Here the LDOS are calculated at the center (i.e., $j = 25$) of the disordered segment with $L = 50$. When $V_{ex} < V_c = 0.92t$, the penetrated Majorana bound states with $\lambda = -1$ and the normal states with $\lambda = 1$ are coupled by the random disordered potentials. As a result, the amplitude of the LDOS at the zero energy remains at a small value. As shown in Fig. 10(b), $\rho^+(j, 0)$ become zero for $V_{ex} > V_c$, which means the all zero-energy states belong to $\lambda = 1$ disappear. As shown in Fig 10(c), $\rho^-(j, 0)$ suddenly becomes large for $V_{ex} > V_c$. All of the penetrated Majorana bound states belong to $\lambda = -1$. As discussed in Sec. IIIC, the number of the zero-energy states at the edge of superconductor is equal to the number of the propagating channels N_c for $V_{ex} > V_c$. In the normal segment, the number of the zero-energy states also becomes N_c even in the presence of disorder because the impurity potentials preserve chiral symmetry. Such Majorana bound states at the zero energy form the resonant transmission channels in the normal segment. This explains the perfect transmission in the presence of disorder.

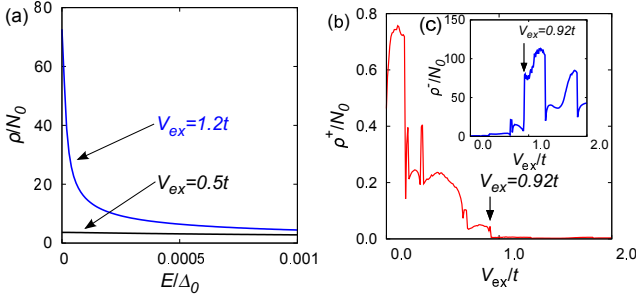


FIG. 10. (Color online) In (a), the LDOS at the center of disordered segment ($j = 25$) is plotted as a function of the energy. We chose $M = 10$, $L = 50$ and $W = 2.0t$. We plot the results for $V_{ex} = 0.5t$ and $1.2t$. The LDOS corresponding to $\lambda = +1$ and the LDOS corresponding to $\lambda = -1$ is plotted as a function of the Zeeman potential in (b) and (c), respectively. The energy for the Green's function is fixed to zero. We chose $M = 10$, $L = 50$ and $W = 2.0t$ in both (b) and (c). The number of samples used for random ensemble average is 10^4 for all the results. The small imaginary part for the Green's function $i\delta$ is chosen as $i10^{-5}\Delta_0$ for all the results.

C. Odd-frequency Cooper pairs

In the normal segment in Fig. 4(a), the zero energy states consist of two contributions: one is the penetrated MBSs from the superconductor and the other is the usual metallic states at the Fermi level. The LDOS in Fig. 9(a) indicates that the former is much dominant than the latter for $V_{ex} > V_c$. According to the argument in Sec. III B, such MBSs in the normal segment belong to $\lambda = -1$. Therefore the anomalous Green's function can be represented by F^- . By applying the analytic continuation, we obtain

$$\hat{\mathcal{F}}^-(\mathbf{r}, \mathbf{r}'; i\omega_n) = \frac{1}{4\omega_n} \sum_{\alpha, \nu_0} \begin{bmatrix} \alpha a_{-, \alpha, \nu_0}(\mathbf{r}) a_{-, \alpha, \nu_0}(\mathbf{r}') & a_{-, \alpha, \nu_0}(\mathbf{r}) a_{-, \alpha, \nu_0}^*(\mathbf{r}') \\ a_{-, \alpha, \nu_0}^*(\mathbf{r}) a_{-, \alpha, \nu_0}(\mathbf{r}') & \alpha a_{-, \alpha, \nu_0}^*(\mathbf{r}) a_{-, \alpha, \nu_0}^*(\mathbf{r}') \end{bmatrix}.$$

The anomalous function satisfies

$$\hat{\mathcal{F}}^-(\mathbf{r}, \mathbf{r}'; i\omega_n) = \left[\hat{\mathcal{F}}^-(\mathbf{r}', \mathbf{r}; i\omega_n) \right]^T. \quad (65)$$

Thus the pairing correlation is spin-triplet even-parity. As a result, the pairing correlation is the odd function of ω_n . Therefore Majorana fermions always accompany the odd-frequency Cooper pairs³². The odd-frequency Cooper pairs support the quantization of the zero-bias conductance in Sec. II B^{28,30}. The Josephson current shown in Sec. II C is carried by the odd-frequency Cooper pairs²⁹.

V. CONCLUSION

We have studied transport properties in junctions consisting of a superconducting nanowire with Dresselhaus[110] spin-orbit coupling. The local density of states

at the edge of the isolated nanowire shows the large zero-energy peak when the Zeeman potential is larger than a critical value. This single peak structure reflects the existence of the Majorana bound states. We show that the number of such Majorana bound states is equal to the number of the propagating channels N_c . When we attach a normal nanowire to the superconducting one, the Majorana bound states penetrate into the normal segment and form N_c resonant transmission channels there. All of the Majorana bound states remains at the zero energy because of chiral symmetry of the junction. As a result, the Majorana bound states in the normal segment are responsible for the perfect transmission. We numerically show that the zero-bias differential conductance of the normal-metal/superconductor junction are quantized at $2e^2 N_c/h$ irrespective of the disorder. The Josephson current in disordered superconductor/normal-metal/superconductor junctions shows the fractional current-phase relationship $J \propto \sin(\varphi/2)$ at a low temperature. The superconducting nanowires with Dresselhaus[110] spin-orbit coupling are two-dimensional analog of the spin-triplet superconductor in the 'polar' state. Our results indicate a way of detecting Majorana Fermions in experiments.

ACKNOWLEDGMENTS

The authors are grateful to J. D. Sau for useful discussion. This work was partially supported by the "Topological Quantum Phenomena" (Nos. 22103002, 22103005) Grant-in Aid for Scientific Research on Innovative Areas and KAKENHI (No. 26287069) from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan and by the Ministry of Education and Science of the Russian Federation (Grant No. 14Y.26.31.0007).

Appendix A: Zero energy states under chiral symmetry

Here, we briefly summarize the argument in Ref. 26 which shows the important properties of the zero-energy states under the chiral symmetry. We consider the BdG Hamiltonian H which preserves the chiral symmetry

$$\Gamma H \Gamma^{-1} = -H, \quad \Gamma^2 = 1, \quad (A1)$$

where Eq. (A1) is equivalent to

$$[H^2, \Gamma] = 0. \quad (A2)$$

The BdG equation is given by

$$H \varphi_E(\mathbf{r}) = E \varphi_E(\mathbf{r}). \quad (A3)$$

When we consider the eigen equation

$$H^2 \chi_{E^2}(\mathbf{r}) = E^2 \chi_{E^2}(\mathbf{r}), \quad (A4)$$

Eq. (A2) suggest that the states $\chi_{E^2}(\mathbf{r})$ is also the eigen states of Γ at the same time. Since $\Gamma^2 = 1$, we find that the eigen value of Γ is $+1$ or -1 . Namely the eigen equation

$$\Gamma\chi_{E^2\lambda}(\mathbf{r}) = \lambda\chi_{E^2\lambda}(\mathbf{r}), \quad (\text{A5})$$

holds for $\lambda = \pm 1$. By multiplying H to Eq. (A5) from the left side and by using Eq. (A1), we obtain the equation

$$\Gamma H\chi_{E^2\lambda}(\mathbf{r}) = -\lambda H\chi_{E^2\lambda}(\mathbf{r}). \quad (\text{A6})$$

We find that $H\chi_{E^2\lambda}(\mathbf{r})$ is the eigen state of Γ belonging to $-\lambda$. Thus we can connect $\chi_{E^2+}(\mathbf{r})$ and $\chi_{E^2-}(\mathbf{r})$ as

$$H\chi_{E^2\lambda}(\mathbf{r}) = c_{E^2\lambda}\chi_{E^2-\lambda}(\mathbf{r}), \quad (\text{A7})$$

where $c_{E^2\lambda}$ is a constant. As shown in Ref. 26, the one-to-one correspondence exists between $\varphi_E(\mathbf{r})$ and $\chi_{E^2}(\mathbf{r})$.

At first, we consider zero energy states $\chi_{0\lambda}(\mathbf{r})$ which satisfies

$$H^2\chi_{0\lambda}(\mathbf{r}) = 0, \quad (\text{A8})$$

in Eq. (A4). The integration of \mathbf{r} after multiplying $\chi_{0\lambda}^\dagger(\mathbf{r})$ from the left results in

$$\int d\mathbf{r} |H\chi_{0\lambda}(\mathbf{r})|^2 = 0. \quad (\text{A9})$$

This means that the norm of $H\chi_{0\lambda}(\mathbf{r})$ is zero. Therefore we conclude that

$$H\chi_{0\lambda}(\mathbf{r}) = 0. \quad (\text{A10})$$

As a result, we find the relation

$$\varphi_{0\lambda}(\mathbf{r}) = \chi_{0\lambda}(\mathbf{r}). \quad (\text{A11})$$

When a zero energy state is described by $\varphi_{0+}(\mathbf{r}) = \chi_{0+}(\mathbf{r})$, the relations in Eqs. (A7) and (A10) suggest that $\chi_{0-}(\mathbf{r}) = 0$. Therefore the zero-energy states are always the eigen states of Γ .

For $E \neq 0$, it is possible to represent $\varphi_E(\mathbf{r})$ by $\chi_{E^2\pm}(\mathbf{r})$ ²⁶. By calculating the norm of $H\chi_{E^2\lambda}(\mathbf{r})$, we obtain

$$E^2 = |c_{E^2\lambda}|^2. \quad (\text{A12})$$

Multiplying H to Eq. (A7) from the left alternatively gives a relation

$$c_{E^2\lambda}c_{E^2-\lambda} = 1. \quad (\text{A13})$$

Therefore, we find the relation

$$H\chi_{E^2\lambda}(\mathbf{r}) = Ee^{i\theta_{E^2}}\chi_{E^2-\lambda}(\mathbf{r}). \quad (\text{A14})$$

Although we cannot fix the phase factor θ_{E^2} , it is possible to express the states $\varphi_E(\mathbf{r})$ for $E \neq 0$ as

$$\varphi_E(\mathbf{r}) = \frac{1}{\sqrt{2}} \left(e^{-i\theta_{E^2}/2} \chi_{E^2+}(\mathbf{r}) + s_E e^{i\theta_{E^2}/2} \chi_{E^2-}(\mathbf{r}) \right), \quad (\text{A15})$$

$$s_E = \begin{cases} 1 & \text{for } E > 0 \\ -1 & \text{for } E < 0. \end{cases} \quad (\text{A16})$$

The nonzero-energy states are constructed by a pair of eigen states for Γ : one belongs to $\lambda = 1$ and the other belongs $\lambda = -1$. Therefore, the states with $E \neq 0$ are not the eigen states of Γ . On the contrary to the nonzero-energy states, the the zero-energy states are the eigen states of Γ .

Appendix B: Description of zero-energy edge states

The BdG Hamiltonian of the Dresselhaus nanowire is represented by \tilde{H}_0 in Eq. (12). By using the unitary matrix

$$\tilde{R} = \begin{bmatrix} \hat{r} & 0 \\ 0 & \hat{r}^* \end{bmatrix}, \quad \hat{r} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-i\pi/4} & -e^{-i\pi/4} \\ e^{i\pi/4} & e^{i\pi/4} \end{bmatrix}, \quad (\text{B1})$$

the BdG Hamiltonian \tilde{H}_0 is first transformed to

$$\begin{aligned} \tilde{H}' &= \tilde{R}\tilde{H}_0\tilde{R}^\dagger \\ &= \begin{bmatrix} \hat{h}' & i\Delta_0\hat{\sigma}_2 \\ -i\Delta_0\hat{\sigma}_0 & -\hat{h}' \end{bmatrix}, \end{aligned} \quad (\text{B2})$$

$$\hat{h}' = \xi\hat{\sigma}_0 + V_{ex}\hat{\sigma}_3 + i\lambda_D\partial_x\hat{\sigma}_2. \quad (\text{B3})$$

The Hamiltonian in this basis is represented only by real numbers. Next we apply a transformation which is similar to the Foldy-Wouthysen transformation³⁹ to the BdG Hamiltonian in Eq. (B2). Using a unitary matrix

$$\tilde{U} = \begin{bmatrix} \hat{u} & 0 \\ 0 & \hat{u} \end{bmatrix}, \quad (\text{B4})$$

$$\hat{u} = \exp[i\hat{S}], \quad \hat{S} = \frac{\lambda_D}{2\hbar V_{ex}} p_x \hat{\sigma}_1, \quad (\text{B5})$$

with $p_x = -i\hbar\partial_x$, we transform H' into

$$\tilde{U}\tilde{H}'\tilde{U}^\dagger = \begin{bmatrix} e^{i\hat{S}}\hat{h}'e^{-i\hat{S}} & e^{i\hat{S}}(i\Delta_0\hat{\sigma}_2)e^{-i\hat{S}} \\ -e^{i\hat{S}}(i\Delta_0\hat{\sigma}_2)e^{-i\hat{S}} & -e^{i\hat{S}}\hat{h}'e^{-i\hat{S}} \end{bmatrix}. \quad (\text{B6})$$

The diagonal term of Eq. (B2) can be expanded as

$$e^{i\hat{S}}\hat{h}'e^{i\hat{S}} = \hat{h}' + i[\hat{S}, \hat{h}'] + \frac{i^2}{2!}[\hat{S}, [\hat{S}, \hat{h}']] + \dots, \quad (\text{B7})$$

with using the Baker-Housdorff formula. We assume large enough Zeeman potential so that $\lambda_D k_F \ll V_{ex}$ is satisfied where $k_F = \sqrt{2m\mu}/\hbar$ denotes Fermi wave number. From this assumption, we obtain

$$e^{i\hat{S}}\hat{h}'e^{i\hat{S}} = \xi\hat{\sigma}_0 + V_{ex}\hat{\sigma}_3 + O\left[\left(\frac{\lambda_D k_F}{V_{ex}}\right)^2\right], \quad (\text{B8})$$

within the first order of $\lambda_D k_F/V_{ex}$. The off-diagonal term corresponding to the pair potential is transformed to

$$\begin{aligned} e^{i\hat{S}}(i\Delta_0\hat{\sigma}_2)e^{-i\hat{S}} &= i\Delta_0\hat{\sigma}_2 + i[\hat{S}, i\Delta_0\hat{\sigma}_2] + \dots \\ &= i\Delta_0\hat{\sigma}_2 - i\frac{\lambda_D\Delta_0}{\hbar V_{ex}} p_x \hat{\sigma}_3 + O\left(\frac{(\lambda_D k_F)^2}{V_{ex}^2}\right), \end{aligned} \quad (\text{B9})$$

where we assume the uniform pair potential (i.e., $[p_x, \Delta_0] = 0$). As the result, the BdG Hamiltonian can be written as

$$\tilde{U} \tilde{H}' \tilde{U}^\dagger = \begin{bmatrix} \xi + V_{ex} & 0 & -i \frac{\lambda_D \Delta_0}{\hbar V_{ex}} p_x & \Delta_0 \\ 0 & \xi - V_{ex} & -\Delta_0 & i \frac{\lambda_D \Delta_0}{\hbar V_{ex}} p_x \\ i \frac{\lambda_D \Delta_0}{\hbar V_{ex}} p_x & -\Delta_0 & -\xi - V_{ex} & 0 \\ \Delta_0 & -i \frac{\lambda_D \Delta_0}{\hbar V_{ex}} p_x & 0 & -\xi + V_{ex} \end{bmatrix} + O\left(\frac{(\lambda_D k_F)^2}{V_{ex}^2}\right). \quad (\text{B10})$$

By interchanging the second column and the third one, and by interchanging the second row and the third one, the Hamiltonian can be deformed as

$$\tilde{H}_{\text{eff}} = \tilde{H}_{p_x} + \tilde{V}_\Delta, \quad (\text{B11})$$

$$\tilde{H}_{p_x} = \begin{bmatrix} \hat{h}_{p_x, \uparrow} & 0 \\ 0 & \hat{h}_{p_x, \downarrow} \end{bmatrix}, \quad (\text{B12})$$

$$\hat{h}_{p_x, \sigma} = \begin{bmatrix} \xi + s_s V_{ex} & -s_s i \frac{\lambda_D \Delta_0}{\hbar V_{ex}} p_x \\ s_s i \frac{\lambda_D \Delta_0}{\hbar V_{ex}} p_x & -\xi - s_s V_{ex} \end{bmatrix}, \quad (\text{B13})$$

$$\tilde{V}_\Delta = \begin{bmatrix} 0 & i \Delta_0 \hat{\sigma}_2 \\ -i \Delta_0 \hat{\sigma}_2 & 0 \end{bmatrix}, \quad (\text{B14})$$

$$s_s = \begin{cases} 1 & \text{for } \sigma = \uparrow \\ -1 & \text{for } \sigma = \downarrow \end{cases}. \quad (\text{B15})$$

where the diagonal components $\hat{h}_{p_x, \sigma}$ are equivalent to the Hamiltonian of the spin-triplet p_x -wave superconductor. Therefore, Eq. (B11) corresponds to the BdG Hamiltonian of the spin-full p_x -wave superconductor \tilde{H}_{p_x} with the spin-mixing term \tilde{V}_Δ . In addition, we find that the BdG Hamiltonian \tilde{H}_{eff} preserves chiral symmetry

$$\Gamma \tilde{H}_{\text{eff}} \Gamma^{-1} = -\tilde{H}_{\text{eff}}, \quad \Gamma = \begin{bmatrix} \hat{\sigma}_1 & 0 \\ 0 & \hat{\sigma}_1 \end{bmatrix}. \quad (\text{B16})$$

Now we seek the wave function of the zero-energy edge states in \tilde{H}_{eff} . We first neglect the spin-mixing term \tilde{V}_Δ . After solving the BdG equation at the zero-energy with $\tilde{V}_\Delta = 0$

$$\tilde{H}_{p_x} \varphi_{\nu_0}(\mathbf{r}) = 0, \quad (\text{B17})$$

we will discuss effects of \tilde{V}_Δ . As shown in Appendix A, the zero-energy states under chiral symmetry is also the eigen states of Γ

$$\Gamma \varphi_{\nu_0 \lambda}(\mathbf{r}) = \lambda \varphi_{\nu_0 \lambda}(\mathbf{r}), \quad (\text{B18})$$

where $\lambda = \pm 1$. From Eq. (B18), the zero-energy states can be written as

$$\varphi_{\nu_0 \lambda}(\mathbf{r}) = [u_{\nu_0 \lambda, \uparrow}(\mathbf{r}), \lambda u_{\nu_0 \lambda, \uparrow}(\mathbf{r}), u_{\nu_0 \lambda, \downarrow}(\mathbf{r}), \lambda u_{\nu_0 \lambda, \downarrow}(\mathbf{r})]^T. \quad (\text{B19})$$

By substituting Eq. (B19) into the BdG equation, we obtain

$$\hat{h}_{p_x, \sigma} \begin{bmatrix} u_{\nu_0 \lambda, \sigma}(\mathbf{r}) \\ \lambda u_{\nu_0 \lambda, \sigma}(\mathbf{r}) \end{bmatrix} = 0, \quad (\text{B20})$$

which is equivalent to

$$\left[\xi + s_s V_{ex} - s_s i \lambda \frac{\lambda_D \Delta_0}{\hbar V_{ex}} p_x \right] u_{\nu_0 \lambda, \sigma}(\mathbf{r}) = 0. \quad (\text{B21})$$

When we apply the hard-wall boundary condition in the y direction, the wave function can be expanded as

$$\varphi_{\nu_0 \lambda, \sigma}(\mathbf{r}) = \sqrt{\frac{2}{M}} \sum_{n=1}^M u_{n, \lambda, \sigma}(x) \sin\left(\frac{n\pi}{M} y\right), \quad (\text{B22})$$

where M denotes the width of the nanowire. By substituting Eq. (B22) into Eq. (B21), we obtain the equations for each transmission channel n ,

$$\left[\partial_x^2 - s_s 2 \frac{\lambda}{\xi_D} \partial_x + k_{n, \sigma}^2 \right] u_{n, \lambda, \sigma}(x) = 0, \quad (\text{B23})$$

where

$$\xi_D = \frac{\hbar^2 V_{ex}}{m \lambda_D \Delta_0}, \quad (\text{B24})$$

$$k_{n, \sigma} = \frac{\sqrt{\mu_n - s_s V_{ex}}}{\hbar}, \quad \mu_n = \mu - \frac{\hbar^2}{2m} \left(\frac{n\pi}{M}\right)^2. \quad (\text{B25})$$

In what follows, we consider the nanowire with the length $2L$ in the x direction (i.e., $-L \leq x \leq L$) and apply the hard-wall boundary condition at the edge of the nanowire,

$$u_{n, \lambda, \sigma}(-L) = u_{n, \lambda, \sigma}(L) = 0. \quad (\text{B26})$$

The length of the nanowire is long enough so that $L/\xi_D \gg 1$ is satisfied. For $\mu_n < V_{ex}$, there is no solution which satisfies the boundary conditions in Eq. (B26).

For $-V_{ex} < \mu_n < V_{ex}$, we find the two solutions in spin-down sector as

$$\varphi_{n, \downarrow}^L(x) = \frac{C_L}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \sin \left[\sqrt{k_{n, \downarrow}^2 - \xi_D^{-2}} (x + L) \right] e^{-x/\xi_D}, \quad (\text{B27})$$

$$\varphi_{n, \downarrow}^R(x) = \frac{C_R}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \sin \left[\sqrt{k_{n, \downarrow}^2 - \xi_D^{-2}} (x - L) \right] e^{x/\xi_D}, \quad (\text{B28})$$

where C_L and C_R are the normalization coefficients. It is easy to show that $\varphi_n^L(x)$ localizing at the left edge belongs to $\lambda = -1$ and $\varphi_n^R(x)$ localizing at the right edge belongs to $\lambda = 1$. In the spin-up sector, on the other hand, there is no solution.

For $\mu_n > V_{ex}$, there are solutions at either edges in both spin-up and spin-down sectors. For the left edge, we find

$$\varphi_{n, \uparrow}^L(x) = \frac{C'_L}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \sin \left[\sqrt{k_{n, \uparrow}^2 - \xi_D^{-2}} (x + L) \right] e^{-x/\xi_D}, \quad (\text{B29})$$

in addition to $\varphi_{n,\downarrow}^L(x)$ in Eq. (B27). The solution $\varphi_{n,\uparrow}^L(x)$ belongs to $\lambda = 1$. The two zero-energy states at the left edge (i.e., $\varphi_{n,\uparrow}^L(x)$ and $\varphi_{n,\downarrow}^L(x)$) belong to the opposite spin sector and to the opposite λ to each other. At the right edge, we find

$$\varphi_{n,\uparrow}^R(x) = \frac{C'_R}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \sin \left[\sqrt{k_{n,\uparrow}^2 - \xi_D^{-2}}(x - L) \right] e^{x/\xi_D} \quad (\text{B30})$$

in addition to $\varphi_{n,\downarrow}^R(x)$ in Eq. (B28). The two zero-energy states at the right edge (i.e., $\varphi_{n,\uparrow}^R(x)$ and $\varphi_{n,\downarrow}^R(x)$) belong to the opposite spin sector and to the opposite λ to each other.

Next, we consider the effects of the spin-mixing term \tilde{V}_Δ in Eq. (B11). For $\mu_n > V_{ex}$, the two zero-energy states exist for each propagating channel at each edge. At the left edge, for example, the spin-up state with $\lambda = 1$ and the spin-down state with $\lambda = -1$ coexist. The spin-mixing term \tilde{V}_Δ couples the two states. As a result, the energy of the coupled states shifts away from the zero-energy because the linear combination of a state with $\lambda = 1$ and a state with $\lambda = -1$ can form the nonzero-energy state as discussed in Eq. (A15). In this case, all the edge states can leave away from the zero-energy. For $-V_{ex} < \mu_n < V_{ex}$, on the other hand, the only one zero-energy state with spin-down exists for each propagating channel at each edge because the zero-energy state with spin-up is absent. The edge states remain at the zero-energy even in the presence of \tilde{V}_Δ because coupling partners are absent. In other words, \tilde{V}_Δ does not affect the zero-energy edge states.

Appendix C: Effective p_x -wave superconductor with coexistence of Rashba and Dresselhaus[100] spin-orbit coupling

We propose an alternative nanowire whose Hamiltonian is unitary equivalent to Eq. (12). Let us consider the two dimensional electron system where the Rashba spin-orbit coupling and the Dresselhaus[100] one coexist. The Hamiltonian is represented by

$$\hat{h}_{RD} = \hat{h}_{\text{kin}} + \hat{h}_R + \hat{h}_D^{100} + \hat{h}_V, \quad (\text{C1})$$

$$\hat{h}_{\text{kin}} = \frac{1}{2m} (p_x^2 + p_y^2) - \mu, \quad (\text{C2})$$

$$\hat{h}_R = \frac{\alpha}{\hbar} (p_y \hat{\sigma}_1 - p_x \hat{\sigma}_2), \quad (\text{C3})$$

$$\hat{h}_D^{100} = \frac{\beta}{\hbar} (p_x \hat{\sigma}_1 - p_y \hat{\sigma}_2), \quad (\text{C4})$$

$$\hat{h}_V = -\mathbf{V} \cdot \hat{\boldsymbol{\sigma}}, \quad \mathbf{V} = V_x \mathbf{e}_1 + V_y \mathbf{e}_2, \quad (\text{C5})$$

where m , μ , α and β denotes the effective mass of an electron, the chemical potential, the strength of the Rashba spin-orbit coupling and the strength of the Dresselhaus[100] spin-orbit coupling, respectively. The unit

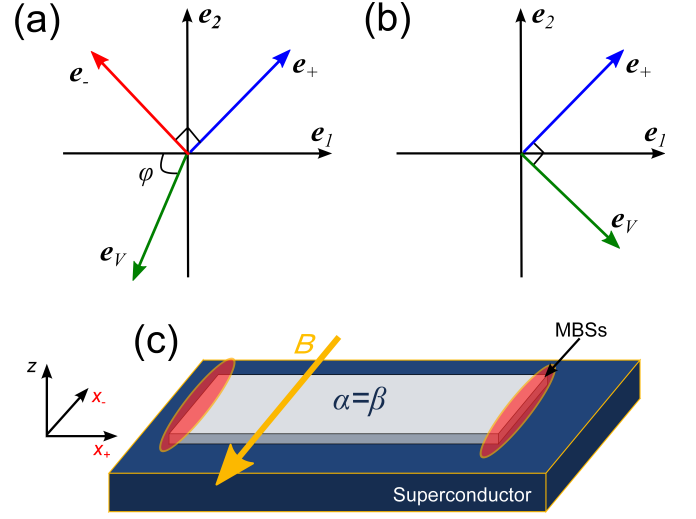


FIG. 11. (Color online) (a) The directions \mathbf{e}_+ and \mathbf{e}_- are orthogonal to each other in the spin-space. The direction of \mathbf{V} is denoted by \mathbf{e}_V . (b) At $\alpha = \beta'$, the spin-orbit interaction proportional to $\hat{\sigma}_-$ vanishes. When we chose the Zeeman potential as $V_x = -V_y$, \mathbf{e}_+ becomes perpendicular to \mathbf{e}_V . (c) Schematic picture of the effective p_x -wave superconductor with the coexistence of the Rashba and Dresselhaus [100] spin-orbit coupling.

vectors in spin space is denoted by \mathbf{e}_j with $j = 1 - 3$. The part of the Hamiltonian \hat{h}_V denotes the Zeeman potential induced by the in-plane external magnetic field. When we define

$$p_{\pm} = \frac{1}{\sqrt{2}} (p_x \pm p_y), \quad (\text{C6})$$

$$x_{\pm} = \frac{1}{\sqrt{2}} (x \pm y), \quad (\text{C7})$$

the commutation relations among p_{\pm} and x_{\pm} become

$$[x_{\nu}, p_{\nu}] = i\hbar\delta_{\nu,\nu}, \quad (\text{C8})$$

$$[x_{\nu}, x_{\nu}] = [p_{\nu}, p_{\nu}] = 0. \quad (\text{C9})$$

In this basis, the Hamiltonian is represented as

$$\hat{h}_{RD} = \hat{h}'_{\text{kin}} + \hat{h}_+ + \hat{h}_- + \hat{h}_V, \quad (\text{C10})$$

where

$$\hat{h}'_{\text{kin}} = \frac{1}{2m} (p_+^2 + p_-^2) - \mu, \quad (\text{C11})$$

$$\hat{h}_{\pm} = \lambda_{\pm} p_{\pm} \hat{\sigma}_{\pm}, \quad (\text{C12})$$

$$\lambda_{\pm} = \frac{1}{\hbar} (\alpha \pm \beta), \quad \hat{\sigma}_{\pm} = \frac{1}{\sqrt{2}} (\hat{\sigma}_2 \pm \hat{\sigma}_1). \quad (\text{C13})$$

As shown in Fig. 11(a), the direction $\mathbf{e}_+ = (\mathbf{e}_1 + \mathbf{e}_2)/\sqrt{2}$ and $\mathbf{e}_- = (\mathbf{e}_1 - \mathbf{e}_2)/\sqrt{2}$ are orthogonal to each other in spin space. When the strength of the Rashba spin-orbit coupling α and the strength of the Dresselhaus spin-orbit

coupling β are equal to each other (i.e., $\alpha = \beta$), λ_- becomes zero. Such electron systems have been studied in spintronics because they show unusual spin property so called spin-helix⁴⁰. In addition, we tune the Zeeman potential as $V_x = -V_y = V/\sqrt{2}$. As shown in Fig. 11(b), the Hamiltonian is constructed by the two orthogonal components in the spin space,

$$\hat{h}_{RD} = \hat{h}'_{\text{kin}} + \lambda_+ p_+ \hat{\sigma}_+ + V \hat{\sigma}_-. \quad (\text{C14})$$

By multiplying a unitary matrix,

$$\hat{d} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-i\pi/8} & ie^{i\pi/8} \\ ie^{-i\pi/8} & e^{i\pi/8} \end{bmatrix}, \quad (\text{C15})$$

the Hamiltonian is transformed into

$$\begin{aligned} \hat{h}'_{RD} &= \hat{d} \hat{h}_{RD} \hat{d}^\dagger \\ &= \hat{h}'_{\text{kin}} - \lambda_+ p_+ \hat{\sigma}_3 - V \hat{\sigma}_1. \end{aligned} \quad (\text{C16})$$

The Hamiltonian \hat{h}'_{RD} is equivalent to the Hamiltonian with the Dresselhaus [110] spin-orbit coupling and with the in-plane magnetic field.

Next, we introduce the proximity-induced s -wave pair potential. The BdG Hamiltonian for the original basis in Eq. (C5) is given by

$$\check{H}_{RD} = \begin{bmatrix} \hat{h}_{RD} & i\Delta_0 \hat{\sigma}_2 \\ -i\Delta_0 \hat{\sigma}_0 & -\hat{h}_{RD}^* \end{bmatrix}. \quad (\text{C17})$$

When $\alpha = \beta$ and $V_x = -V_y = V/\sqrt{2}$, the BdG Hamiltonian is transformed into

$$\begin{aligned} \check{H}'_{RD} &= \hat{D} \check{H}_{RD} \hat{D}^\dagger \\ &= \begin{bmatrix} \hat{h}'_{RD} & i\Delta_0 \hat{\sigma}_2 \\ -i\Delta_0 \hat{\sigma}_0 & -\hat{h}'_{RD}^* \end{bmatrix}, \end{aligned} \quad (\text{C18})$$

$$\hat{D} = \begin{bmatrix} \hat{d} & 0 \\ 0 & \hat{d}^* \end{bmatrix}. \quad (\text{C19})$$

The BdG Hamiltonian \check{H}'_{RD} is unitary equivalent to the BdG Hamiltonian in Eq. (12). Therefore, by applying the unitary transformations introduced in Appendix B, we can obtain the BdG Hamiltonian of the effective p_x -wave superconductor in Eq. (14). As shown in Fig. 11(c), we have to prepare the nanowire along the x_+ direction and apply the magnetic field in the x_- direction. More generally, by applying the appropriate unitary transformation, we also obtain the Hamiltonian \check{H}'_{RD} with

$$\mathbf{V} = V \left[\frac{\sin \phi}{\sqrt{2}} \mathbf{e}_1 - \frac{\sin \phi}{\sqrt{2}} \mathbf{e}_2 + \cos \phi \mathbf{e}_3 \right], \quad (\text{C20})$$

where ϕ is the arbitrary angle of the magnetic field. Alternatively, it is possible to reach BdG Hamiltonian in Eq. (12) when we prepare the nanowire along the x_- direction with $\alpha = -\beta$ and

$$\mathbf{V} = V \left[\frac{\sin \phi}{\sqrt{2}} \mathbf{e}_1 + \frac{\sin \phi}{\sqrt{2}} \mathbf{e}_2 + \cos \phi \mathbf{e}_3 \right]. \quad (\text{C21})$$

-
- ¹ E. Majorana, Nuovo Cimento, **14**, 171 (1937).
² F. Wiczek, Nature Phys. **5**, 614 (2009).
³ D. A. Ivanov, Phys. Rev. Lett. **86**, 268 (2001).
⁴ J. D. Sau, D. J. Clarke, and S. Tewari, Phys. Rev. B **84**, 094505 (2011).
⁵ N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
⁶ A. Y. Kitaev, Phys. Usp. **44**, 131 (2001).
⁷ L. Fu and C. L. Kane, Phys. Rev. Lett. **100**, 096407 (2008).
⁸ M. Sato, Y. Takahashi, and S. Fujimoto, Phys. Rev. Lett. **103**, 020401 (2009).
⁹ J. D. Sau, R. M. Lutchyn, S. Tewari, and S. DasSarma, Phys. Rev. Lett. **104**, 040502 (2010).
¹⁰ J. Alicea, Phys. Rev. B **81**, 125318 (2010).
¹¹ R. M. Lutchyn, J. D. Sau, and S. DasSarma, Phys. Rev. Lett. **105**, 077001 (2010).
¹² Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. **105**, 177002 (2010).
¹³ T. D. Stanescu, R. M. Lutchyn, and S. DasSarma, Phys. Rev. B **84**, 144522 (2011).
¹⁴ M. Sato and S. Fujimoto, Phys. Rev. B **79**, 094504 (2009).
¹⁵ S. Sasaki, M. Kriener, K. Segawa, K. Yada, Y. Tanaka, M. Sato, and Y. Ando, Phys. Rev. Lett. **107**, 217001 (2011).
¹⁶ A. C. Potter and P. A. Lee, Phys. Rev. Lett. **105**, 227003 (2010); A. C. Potter and P. A. Lee, Phys. Rev. B **83**, 094525 (2011).
¹⁷ R. M. Lutchyn, T. D. Stanescu, and S. DasSarma, Phys. Rev. Lett. **106**, 127001 (2011).
¹⁸ M. Gibertini, F. Taddei, M. Polini, and R. Fazio, Phys. Rev. B **85**, 144525 (2012).
¹⁹ G. Dresselhaus, Phys. Rev. **100**, 580-586 (1955).
²⁰ J. You, C. H. Oh, and V. Vedral, Phys. Rev. B **87**, 054501 (2013).
²¹ A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).
²² L. J. Buchholtz and G. Zwicknagl, Phys. Rev. B **23**, 5788 (1981).
²³ J. Hara and K. Nagai, Prog. Theor. Phys. **74**, 1237 (1986).
²⁴ C. R. Hu, Phys. Rev. Lett. **72**, 1526 (1994).
²⁵ Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995).
²⁶ M. Sato, Y. Tanaka, K. Yada, and T. Yokoyama, Phys. Rev. B **83**, 224511 (2011).
²⁷ Y. Tanaka, Y. Asano, A. A. Golubov, and S. Kashiwaya, Phys. Rev. B **72**, 140503(R) (2005).
²⁸ Y. Tanaka and S. Kashiwaya, Phys. Rev. B **70**, 012507 (2004).
²⁹ Y. Asano, Y. Tanaka, and S. Kashiwaya, Phys. Rev. Lett. **96**, 097007 (2006).
³⁰ Y. Asano, Y. Tanaka, A. A. Golubov, and S. Kashiwaya, Phys. Rev. Lett. **99**, 067005 (2007).
³¹ Y. Asano, A. A. Golubov, Ya. V. Fominov, and Y. Tanaka, Phys. Rev. Lett. **107**, 087001 (2011).
³² Y. Asano and Y. Tanaka, Phys. Rev. B **87**, 104513 (2013).

- ³³ Y. Tanaka, S. Kashiwaya, and T. Yokoyama, Phys. Rev. B **71**, 094513 (2005).
- ³⁴ P. A. Lee and D. S. Fisher, Phys. Rev. Lett. **47**, 882-885 (1981).
- ³⁵ T. Ando, Phys. Rev. B **44**, 8017-8027 (1991).
- ³⁶ G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).
- ³⁷ Y. Asano, Phys. Rev. B **63**, 052512 (2001).
- ³⁸ K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).
- ³⁹ L. L. Foldy and S. S. Wouthuysen, Phys. Rev. **78**, 29-36 (1950).
- ⁴⁰ B. A. Bernevig, J. Orenstein and S. C. Zhang, Phys. Rev. Lett. **97**, 236601 (2006).